Affine Wealth Model

or Trickle Up

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What is it?

- A simple yet accurate model of wealth distribution

plus

- A rationale for the distribution

It is a type of stochastic agent based Asset Exchange Model (AEM), these were first developed in 1986. The Affine Wealth Model (AWM) was first published in 2018

Affine describes scaling properties that make it easier to calculate. It also sounds more scientific ☺

How unequal is wealth distribution?

Oxfam estimate that 26 families have as much wealth as the lower 50% of the worlds population, 3.5 billion people.

The Forbes 400, the top 0.00012% of the US, holds as much wealth as the bottom 60%.

At the other end of the spectrum 11% of the population of America have ‘negative wealth’. Counting all their assets they still owe money.

Main Sources

I came across it first in Scientific American

November 2019 – Is Inequality Inevitable – Bruce M Boghosian
https://www.scientificamerican.com/article/is-inequality-inevitable/

The Affine Wealth Model: An agent-based model of asset exchange that allows for negative-wealth agents and its empirical validation

Jie Li, Bruce M. Boghosian, Chengli Li
https://arxiv.org/abs/1604.02370

Plus healthy doses of Wikipedia

Only recently came across the following which does this far better than me ;-)

https://www.college-de-france.fr/site/jean-philippe-bouchaud/seminar-2021-05-12-11h00.htm

Before Asset Exchange Models

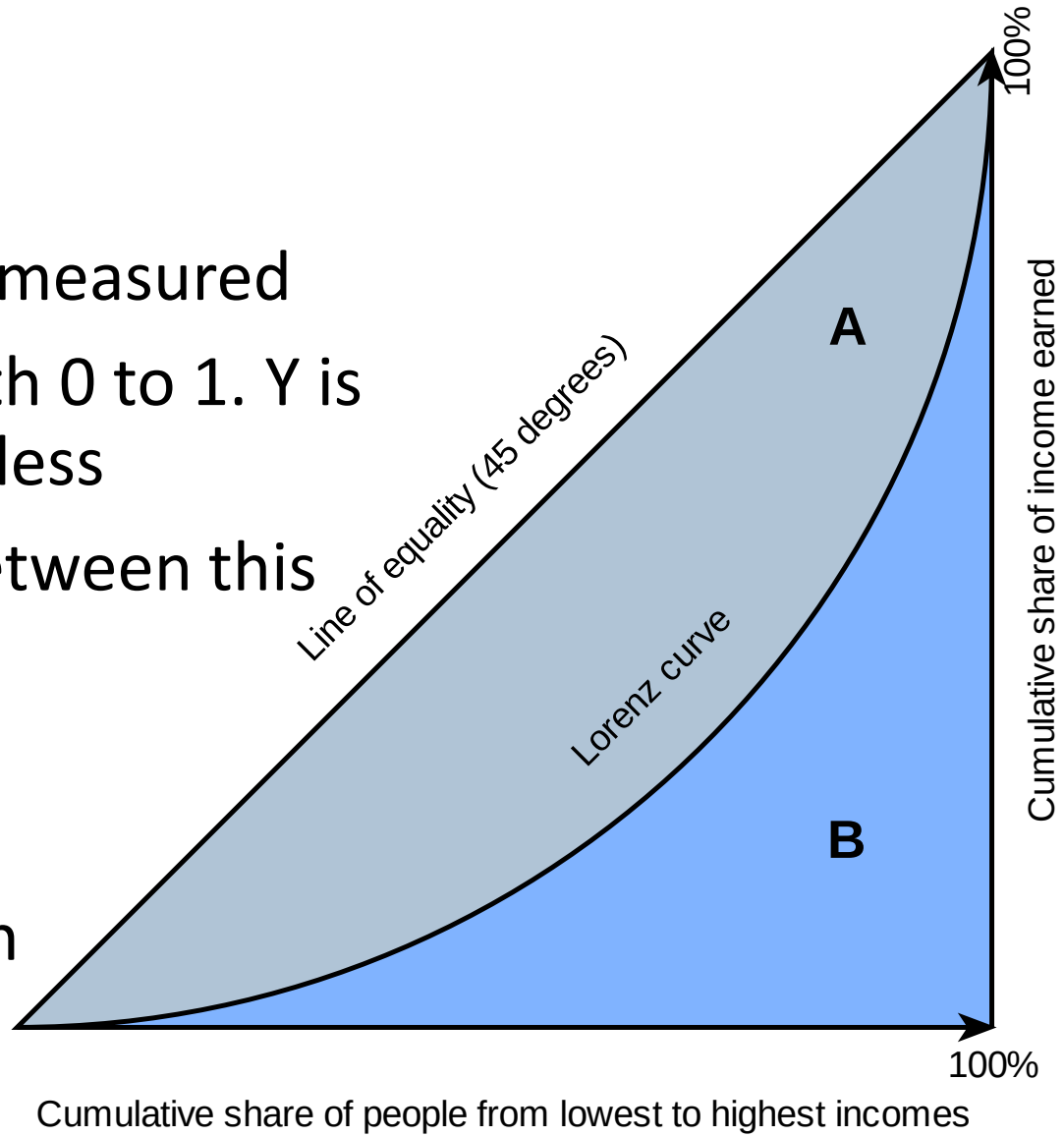
Vifrendo Pareto of the Pareto principle and Pareto efficiency found a good model of how wealth was distributed in 1909 with the Pareto distribution. This can be used to model a wide variety of phenomena like the size of meteorites, insurance losses, and how much different games are played.

Basically it says the proportion of people that have more than wealth x goes down as the power law $\left(\frac{x_m}{x}\right)^{\alpha}$. The index α is called the Pareto index and is one measure of wealth inequality.

This is a pure curve fitting model and has no good rationale.

Gini Coefficient

- How wealth inequality is currently measured
- Lorenz curve: X is cumulative wealth 0 to 1. Y is percentage having that amount or less
- Gini coefficient is twice the area between this curve and the diagonal line of total egalitarianism.
- 0 for totally egalitarian
- 1 for total oligarchy – a tiny fraction holding all the wealth.



The Yard Sale Model (YSM)

The first asset exchange model in 1986.

A number of actors have a certain amount of wealth each.

Two actors A and B are chosen at random, suppose A is poorer with wealth a. A is only willing to risk a percentage of their wealth.

A fair coin is tossed and A can win pa from B or lose qa to B where p and q are small fractions.

We would expect that if $p > q$ then this is good for A, but that's not necessarily so with repeated exchanges.

What happens with Yard Sale Model?

Can run a simulation at http://www.physics.umd.edu/hep/drew/math_general/yard_sale.html

Its default is 100 people, p=20% q=17%. There was a bit of to and fro but after 80000 exchanges a single person had practically all the money. With q=16% however this does not happen, the wealth moves around randomly.

This basic reason for this is (or should be) well known to investors and is the basis for the 'Kelly criterion'. One should use the geometric average rather than the arithmetic average for the gain when engaged in a number for interactions like this.

Average gain in Yard Sale Model

For p=20% and q=17% the gain per exchange is $\sqrt{1.20 * 0.83} = 0.998$

After 1000 exchanges the proportion of the original wealth one would expect is $0.998^{1000} = 0.135$. a bit less than a seventh of the original amount. With q=16% one would expect 54 times as much as one started with. This is assuming one has less wealth than the other one every time.

If the expectation is to lose but the total wealth is constant it has to go somewhere. To the ‘oligarch’ which in this case is a single person.

If everyone gains then nobody tends towards zero wealth so it is spread around.

A side note

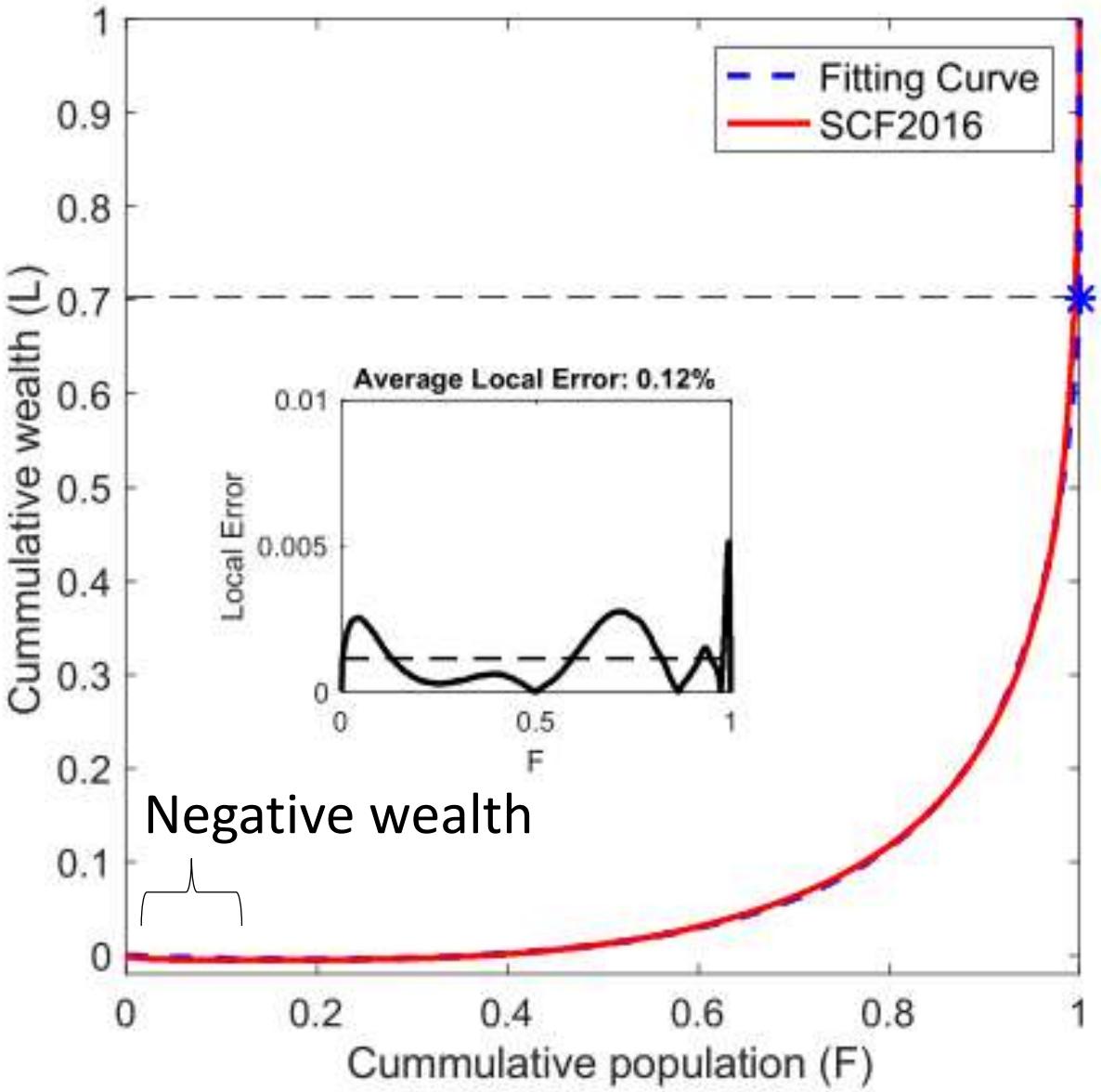
Actually the Kelly criterion says that if you have evens of winning 20% on a bet or losing 17% one should bet (0.5/0.17 - 0.5/0.2) = 0.44 of ones money to increase ones wealth as quickly as possible. And we have

$$\sqrt{(1+0.44*0.2)*(1-0.44*0.17)} = 1.0033$$

So instead of gradually losing money we slowly get wealthier. After 1000 bets our pot would on average be thirty times larger rather than a seventh the size.

A bit more more realism

- There should be taxes. “The only two certainties of life are death and taxes.” In fact perhaps we should really think in terms of redistribution to offset the inbuilt bias towards oligarchy.
- Richer people have an advantage. They can afford to wait, they are given better rates, they can afford tax advisers. “Anyone who has ever struggled with poverty knows how extremely expensive it is to be poor” – James Baldwin.
- Many people are in debt, 11% of Americans have ‘negative wealth’. This is fairly easy to model by adjusting the minimum wealth to be negative rather than zero.



(j) Lorenz Curve Fit of SCF2016

Oligarchy

According to the Congressional Budget office Gini in America in 2016 was about 0.42

However it does not include the Forbes 400 or cater for negative wealth.

Gini calculated by authors of AWM is about 0.83

(Survey of Consumer Finances)

‘Simplification’

The Affine Wealth Model has infinitesimal transactions continually being done. The maths then falls within Stochastic Calculus (which I’m still trying to get a good grasp of!) and is used both in statistical mechanics and in mathematical finance.

Examples are Brownian motion and the Black-Sholes equation

Using this gives the final stable Lorenz curves of wealth in the model.

Stochastic Calculus

A quick idea and then will ignore, we'll just use a finite approximation. ☺

Started with Norbert Wiener turning Brownian motion into a continuous process. A drunkard's walk will go plus or minus 1 step every time unit at random to position $X(t,\omega)$. The variance $E[X*X] = \sigma^2$ grows as the number of steps. With unequal steps or probabilities one also has non-zero $E[X] = \mu$.

A continuous version can be constructed as the limit of chopping the time unit into Δt sized bits and stepping $\sigma\sqrt{\Delta t}$ each time with equal probability to keep the same standard deviation plus a drift $\mu\Delta t$.

Approximation to the model

Start with the Yard sale model with equal odds and the same amount each way

Two ‘agents’ selected at random with wealth w and x

$$\Delta w = \sqrt{\gamma \Delta t}\, min(w, x)\, \eta$$
$$\Delta x = -\sqrt{\gamma \Delta t}\, min(w, x)\, \eta$$

γ (gamma) Some scaling constant, can be set to 1

$\eta = \eta(t, \omega)$ (eta) stochastic variable -1 or +1 with equal probability

Add Redistribution rate

Population size N

Total wealth W

Same as previous except every agent gets their amount adjusted towards the mean. In the proper model every agent also gets involved in a random mix of the yard sale model per unit time as well.

$$\Delta w \mathrel{+}= \chi\left(\frac{W}{N} - w\right)$$

χ (chi) is the redistribution rate

Overall this does not change the total wealth.

Add Wealth-Attained Advantage (WAA)

Change η to give an advantage to the wealthier agent.

$$E[\eta] = \zeta \sqrt{\frac{\Delta t}{\gamma}} \left(\frac{w - x}{W/N} \right)$$

ζ (zeta) the WAA coefficient.

ESYM and AWM

So far we have the extended yard sale model (ESYM). Shifting and scaling it allows for negative wealth and gets to the Affine Wealth Model (AWM).

When cast into stochastic calculus and normalized so $\mathrm{W} = \mathrm{N} = \gamma = 1$ the EYSM has only two parameters, χ the the redistribution rate and ζ the wealth attained advantage. And the AWM has only 3 adding μ a shifted wealth.

Easier to just deal with the Extended yard sale without μ as that just scales things.

Limits of my Skill

Actually doing the going to the limit into stochastic calculus needs something called Ito Integration and leads to what is called a Fokker-Planck equation which is used in statistical mechanics. I'm not up to it myself yet and it's not necessary for the talk. And the equation here would take up a lot of not very informative lines.

However much of the result is fairly straightforward ☺

Qualitive results

With this model, if $\chi < \zeta$, redistribution rate is less than the wealth attained advantage, then there is an oligarchy. This is called a supercritical regime and the oligarchy holds $1 - \chi/\zeta$ of the total wealth. If $\chi \geq \zeta$ it is subcritical and there is no oligarchy.

To get to the 'Affine' in the model's name, suppose the distribution of agents by wealth is given by $P(w,\ \chi,\ \zeta)$, then we have the following duality linking the supercritical and subcritical regimes

$$P\left(\frac{\chi}{\zeta}w,\ \chi,\ \zeta\right) = P(w,\ \zeta,\ \chi) \quad \text{if } \chi < \zeta$$

The Lorenz curve and Gini

The Lorenz curve giving the Gini coefficient is the total wealth of agents with wealth up to w divided by the fraction of agents with wealth less than w. Ignoring negative wealth this curve will scale just like P under the duality. With negative wealth a bit more adjustment has to be made but both it and the Gini can be calculated for given parameters.

The duality and the shifting make it much easier to do the inverse - to map a given Lorenz curve onto the model and find what parameters fit best. And the parameters then give a relative measure of the size of the oligarchy and negative wealth as numbers like Gini.

Interestingly in most of Europe χ is just slightly lower than ζ.

Cons?

- Assumption that the agents work randomly whilst humans make decisions.
- They fit the curve rather than worked from economic data.
- The Pareto distribution can describe the distribution well with two parameters, of course one should expect three to do it better.
- Just because it fits well does not mean it is an explanation.
- Wealth is growing overall rather than being fixed
- Doesn’t count that the rich save more
- Surely richer people get their money through greater ability?

Pro

- It is a simple model, and even a spherical cow model can be useful for abstracting the important aspects.
- It is based on premises which have some relation to reality
- It is quite accurate so it might be possible to learn from it.
- It can be used to quantify things like an oligarchy or negative wealth that the Gini coefficient on its own can’t.
- Growing wealth or saving can be added but is not necessary for an accurate model.

Other responses

Seems to annoy some economists!
- “the pseudo-Scientific American”, “physicists”

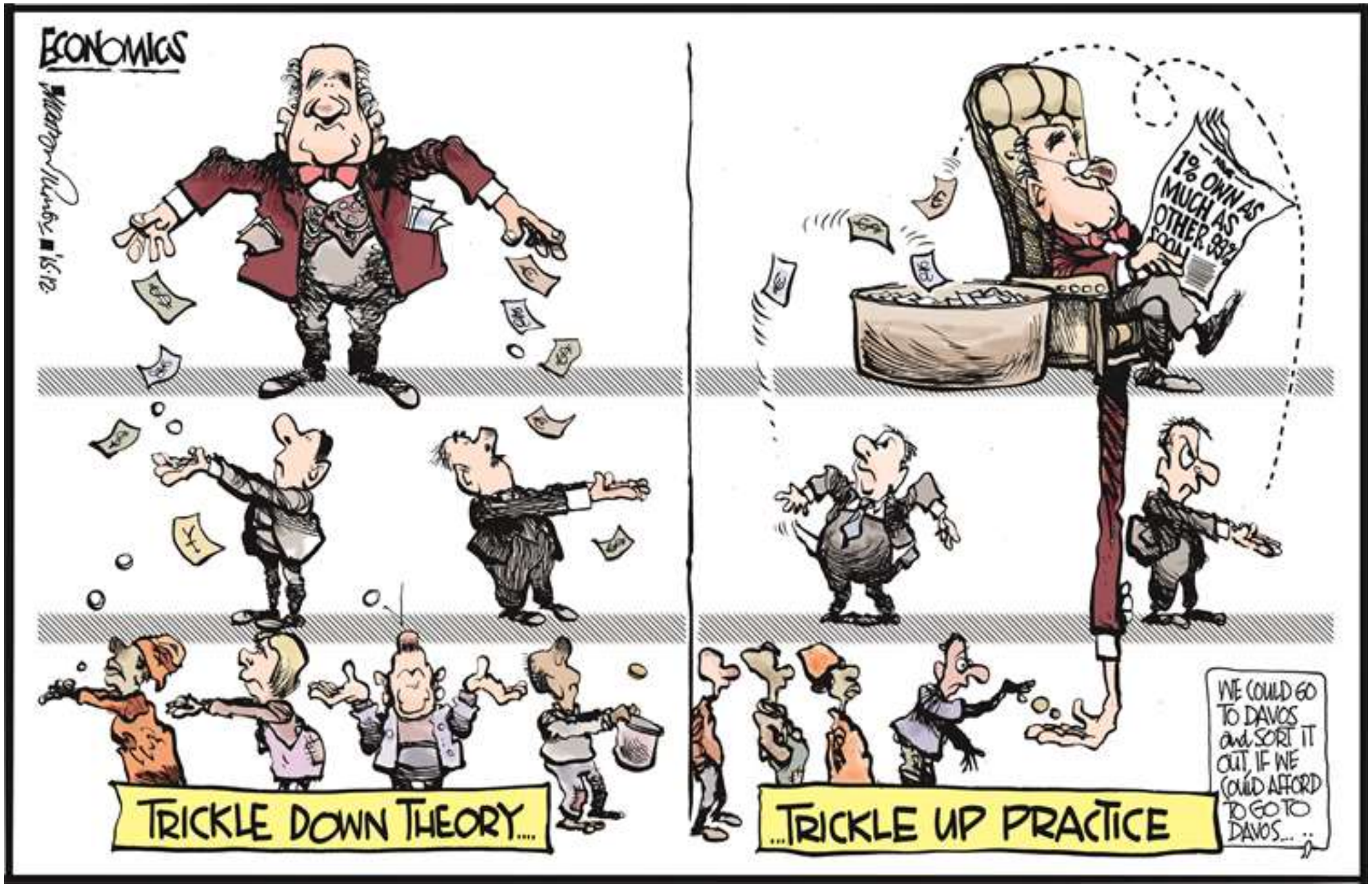
There is a big difference from classical economics – if we followed the doctrine of homo economicus there would be no trades as they would lead to the poor getting poorer. This shows that classical economics needs some revision.

Thoughts on implications

There are good reasons for thinking pure free market economics is very biased and flawed. There is no good reason for thinking the winners or losers have contributed in any remotely proportionate way to their fate.

The word economics comes from the Greek Oikonomia. This translates as household management but also meant the ethical and social use of money.

They had another word chrematistics which describes modern free market economics better, it translates as the study of wealth but also meant its pursuit and unfettered accumulation.


ECONOMICS
TRICKLE DOWN THEORY....
1% OWN AS MUCH AS 99% OF THE REST
...TRICKLE UP PRACTICE
WE COULD GO TO DAVOS and SORT IT OUT, IF WE COULD AFFORD TO GO TO DAVOS....

Seen since

https://www.propublica.org/article/the-secret-irs-files-trove-of-never-before-seen-records-reveal-how-the-wealthiest-avoid-income-tax

Maybe explains the difference in estimates of Gini in America!